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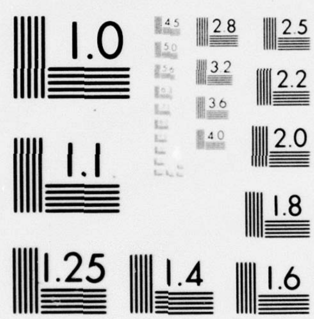

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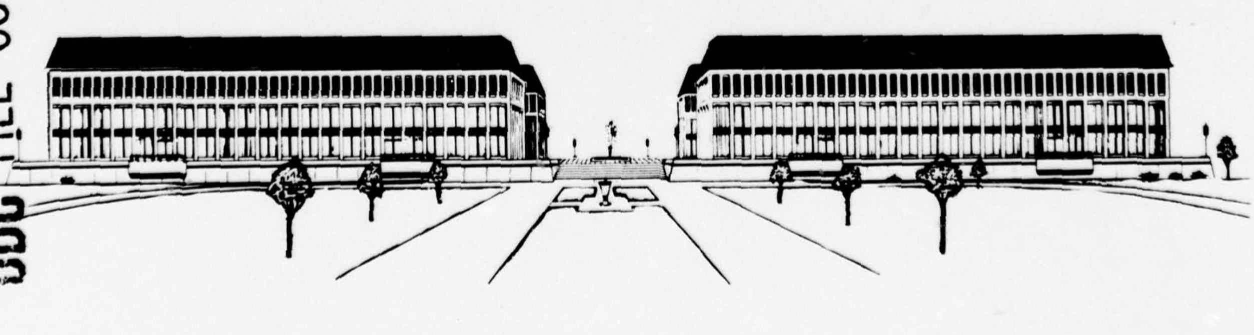

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ERROR ANALYSIS OF CASH METHOD: COMPUTER-AVERAGING OF NOISY PERIODIC SIGNALS

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ABSTRACT

ERROR ANALYSIS OF CASH METHOD: COMPUTER-AVERAGING OF NOISY PERIODIC SIGNALS

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Computer-assisted synchronous hot wire, or "CASH," method was developed to permit computer averaging of hot wire data when sampling oscillating velocity profiles. Using emitted sound pressure as phase reference, computer separates periodic component of hot wire signal from turbulent background by averaging over large number of acoustic cycles. So far technique has been utilized in two investigations: (1) interface oscillations of flow-excited cavities, and (2) growth of instability waves in laminar jets. Accuracy depends on the ability to match closely the reference source frequency. Any mismatch produces systematic error. For given amount of mismatch, however, error is minimized by dividing sample into subsets and performing the averaging at two levels. In the cavity resonator problem, random deviations from periodicity in the reference itself make frequency-matching difficult. Measured standard deviations show a predicted periodicity, with greatest deviation occurring 90° after signal maximum. By counting beats against a standard oscillator, frequency error can be estimated at data acquisition time. With this information, sync error in average signal can be completely removed by post-run data reduction.

Work sponsored by Naval Ship System Command, GHR program, administered by Naval Ship Research and Development Center.

ERROR ANALYSIS OF CASH METHOD: COMPUTER-AVERAGING OF NOISE PERIODIC SIGNALS

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Slide 1.

The computer-assisted synchronous hot wire, or "CASH" method was developed to permit computer averaging of hot wire data when sampling oscillating velocity profiles. In the figure a cavity is shown being excited by fluid flow. A hot wire probe scans the field in the mouth opening, recording velocity data in computer core. Simultaneously the signal from the sound pressure microphone, inside the cavity, is also recorded, to give a phase reference.

Later, the computer data array is averaged to remove turbulence noise, and re-arranged to give the time sequence of vertical profile oscillations in the shear layer.

Slide 2.

The idea is to separate the periodic component of the hot wire signal from background turbulence. Turbulence noise, being random, is easily removed by averaging, but to preserve the phase relation in the pressure signal, the averaging must be done synchronously. Typical initial noise level is 100% of signal, i.e., $S/N \sim 1$. After 10 periods of averaging N/S level is reduced by factor 3, after 100, by factor 10, etc., since noise drops as $1/\sqrt{M}$.

Length of sample run was not extended beyond 100, due to core size limitation.

Slide 3.

In addition to vertical velocity profiles, it is possible to form horizontal profiles of the interface displacement oscillation, by a further reduction of the data.

Here is a sequence of interface wave motions for a cavity oscillating at 283 Hz, taken at 0° (i.e., in-phase with pressure), 90°, 180°, and 270° in the cycle. Flow is to the right at a free stream speed of 27 m/s, with leading edge at left, and trailing edge at right. Circles represent planes at which measurement was performed.

Slide 4.

In order to obtain a meaningful average, the periodic component of the velocity signal is sampled at exactly 20 times the sound frequency for 100 periods of the acoustic signal. In principle this should give a faithful reproduction of the periodic part of the velocity signal, out to the 10th harmonic.

To determine the reliability of the method, the computer was programmed to calculate the normalized standard deviation of the velocity signal during the averaging process. The measured standard deviation when plotted as a function of time, was found to have a large periodic component, suggesting some systemmatic error process involved in the method. A likely source was suspected to be the inability of the experimenter to obtain perfect synchronism between the sound and sampling frequencies.

Slide 5.

Suppose that there is an error Δf in setting the sampling frequency f, then there will be an imperfect phase closure, $\Delta\theta_i$ in each cycle, causing the apparent period to be given by $T'=T(1+\Delta f/f)$ as shown. The measured signal level at phase θ in the ith cycle will be

$$A_i = A_p \cos(\theta-\phi_1+\Delta\theta_i) + \varepsilon$$

where A_p = "True" amplitude of periodic signal

ϕ_1 = initial offset

ε = random noise component

Slide 6.

Computing the average over M-periods, we see that, for small total closure error $\Delta\theta_M$, the effect of sync error cancels out, leaving only the "true" signal after averaging. (The random error, ε, by hypothesis, is taken to average out to zero.)

Slide 7.

From the estimated mean, the variance can be formed, as shown. Again, for small $\Delta\theta_M$, the expression may be simplified somewhat. The residual variance has both periodic and random components.

Slide 8.

The Standard Deviation, normalized by the amplitude of the first Fourier Component of the signal, is evaluated by summing over the geometric series in $\Delta\theta_i$. The periodic component of σ is found to vary with M. (For large M, the variation approaches simple proportionality.)

Slide 9.

Here is what the computer-averaged data and the NSD look like when plotted over one period. Data are represented by circles, while solid lines represent theoretical predictions. The frequency error, Δf, estimated by counting beats between the sound and the standard oscillator signal was found to be about 1.2 Hz.

Note the distinctly periodic nature of the error. Fitting the deviation curve to the theoretical model was done by taking the value at $\theta = \phi_1$ to be the random noise component.

Despite the large deviation near 90° and 270°, the fit to the sampled sound pressure was very close, as predicted. In this run Sound Pressure was analyzed rather than velocity, because its nearly sinusoidal waveform best fitted the theoretical model.

Slide 10.

Before dealing with the velocity data, the more general case must be treated. This is done by expressing the signal as a Discrete Fourier Series of N terms, where N = ½ the number of points sampled per period. Also the requirement of small closure error is dropped. When the average is taken, $\bar{\varepsilon}$ again vanishes by hypothesis, while $\overline{\sin\Delta\theta_j}$ is found to vanish due to symmetry. The $\overline{\cos\Delta\theta_j}$ term turns out to be the familiar single-slit-diffraction function, sin x/x. Since this term remains as a "weighting function" in the average-value series, we see that, in the general case, the effect of sync error does not automatically cancel out!

Slide 11.

Here the error-weighting function is plotted for several values of M.

For a given value of α, (and thus of $\Delta f/f$), least error is produced in the answer by averaging all the data in pairs of cycles, i.e., taking M=2. Unfortunately this requires maximum computer processing time. For practical purposes, a value of M=10 was found to be most satisfactory.

Slide 12.

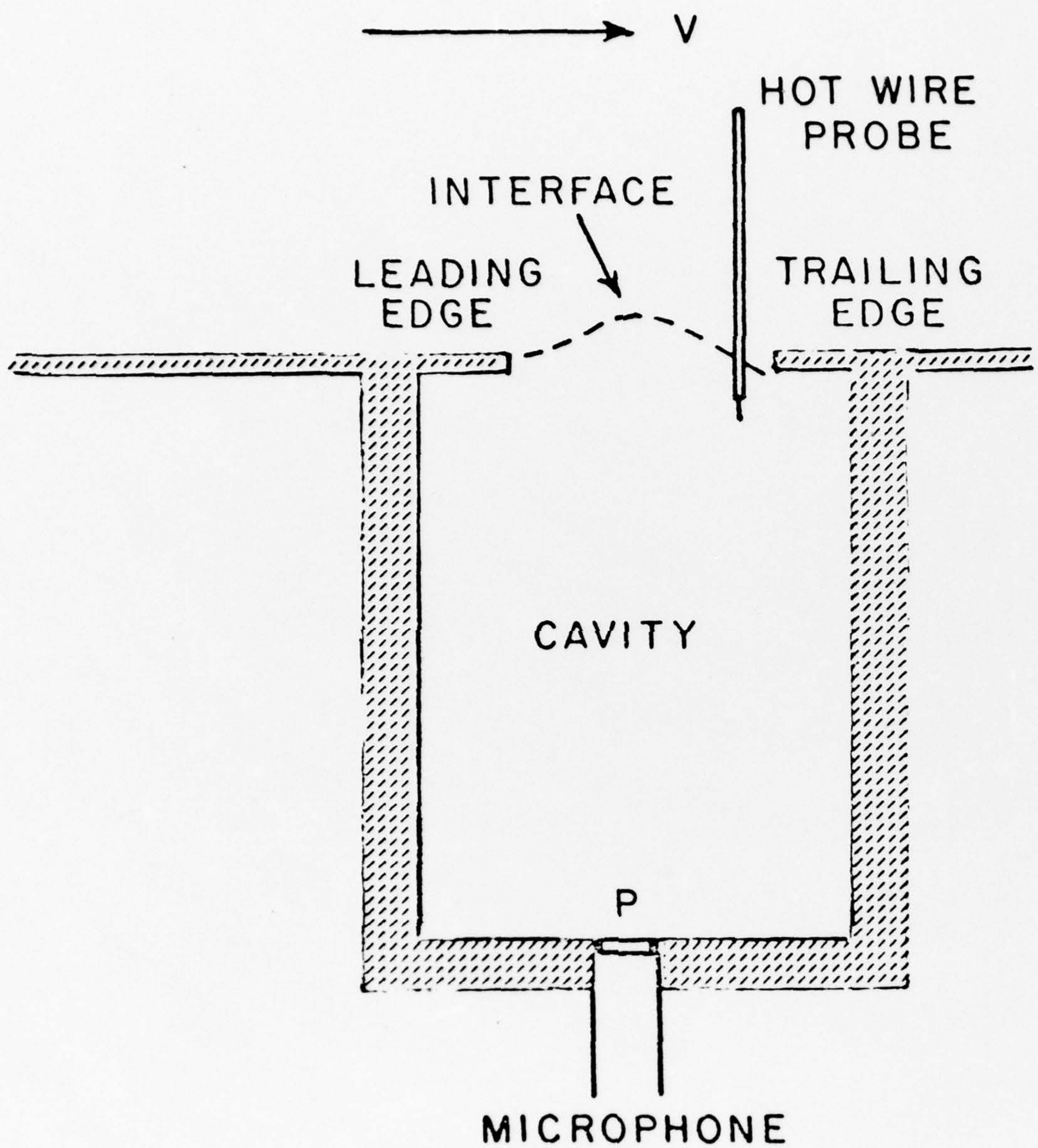
Here is shown the computation of normalized standard deviation for the general case. Despite cancellations, the expression for σ/A_{p1} remains complicated. Dependence on $\Delta f/f$ can be illustrated by performing the average of $\sin^2\Delta\theta_j$ numerically for several values of M.

Slide 13.

The calculations are shown here. It may be seen that, for small $\Delta f/f$, the error rises more quickly as M gets larger. However, the maximum value of $\sin^2\Delta\theta_j$ actually occurs for M = 2.

Slide 14.

From the variation of $\overline{\cos\Delta\theta_j}$ with M and α, one can see why a value of ten is a good choice for M. An arrow shows the abscissa of the highest harmonic for the data analyzed (i.e., the 10th). Sync error for this term is no greater than 15% at M = 10, while for lower harmonics the error is considerably less. Since 100 periods must be averaged to properly reduce the background noise, a choice of M = 10 means that the data must be grouped into ten subsets of ten periods each. If M = 2 had been chosen, the error would have been less, but the computation time would have been increased by a factor of nearly five times, since 50 subsets would be formed. On the other hand, if M = 100 were chosen, performing the average in a single sweep, gross distortion of the signal would result. Since the exact form of the weighting factor is known, an interesting possibility arises that the sync error could be completely removed by a post-run calculation, provided a reliable estimate of sync frequency error is obtained at data acquisition time. (The beat-counting method used previously would be suitable for this.) However, because of the additional computational time that this would introduce, it is more convenient to choose an operating region for which the sync error can be ignored relative to ether experimental errors.



CAVITY RESONATOR

Fig 1

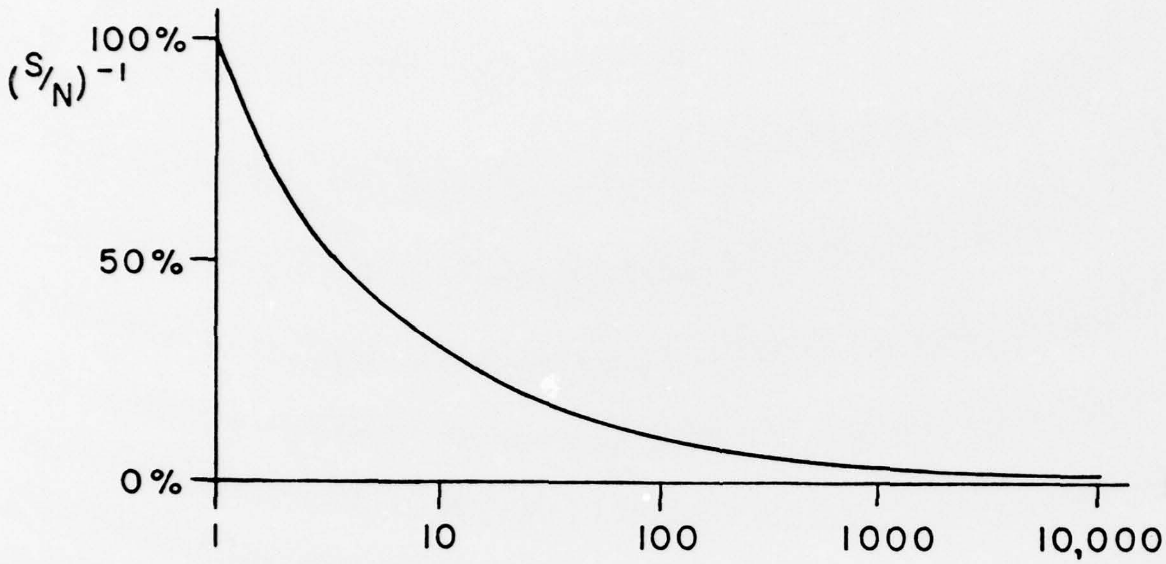
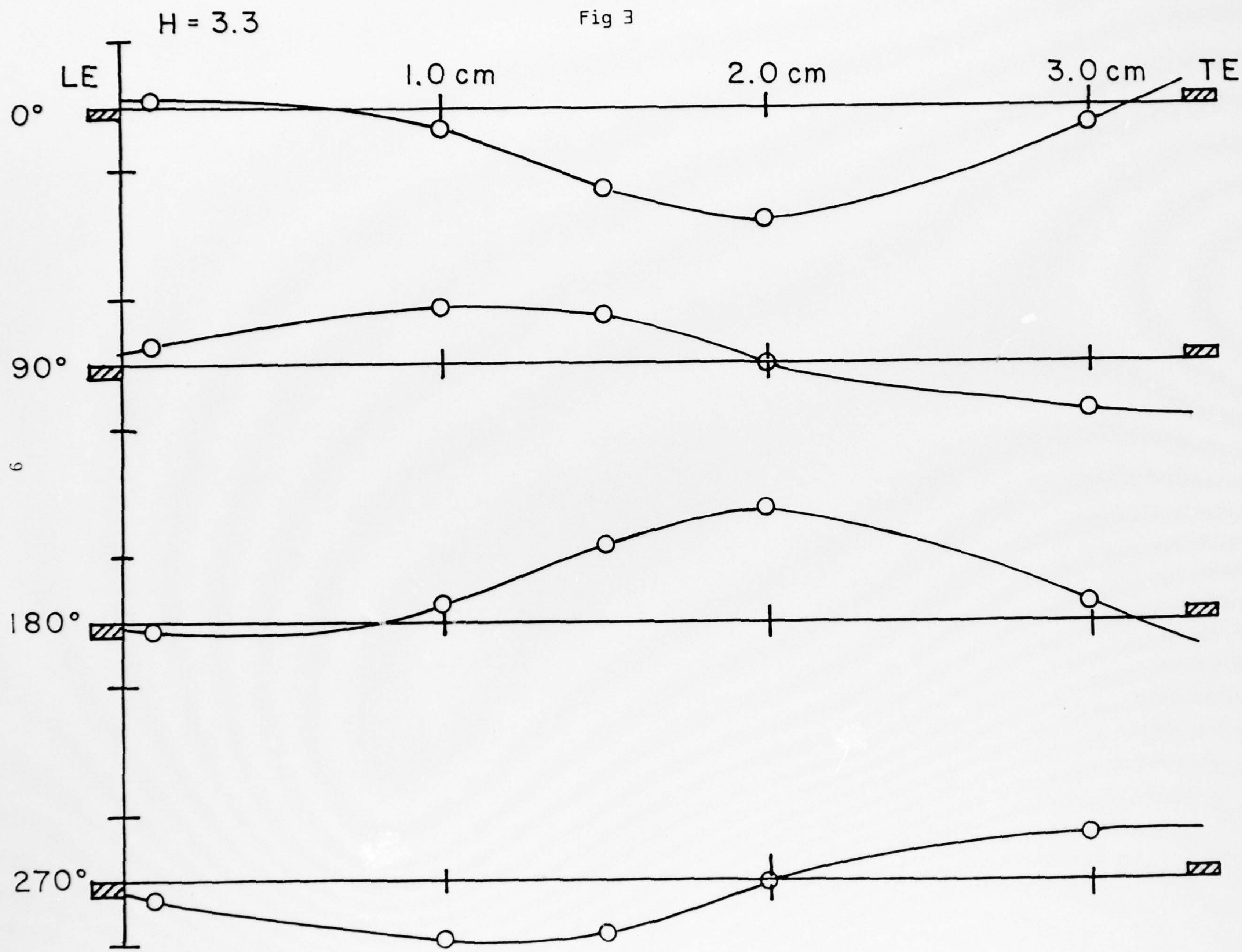


Fig 2

Fig 3



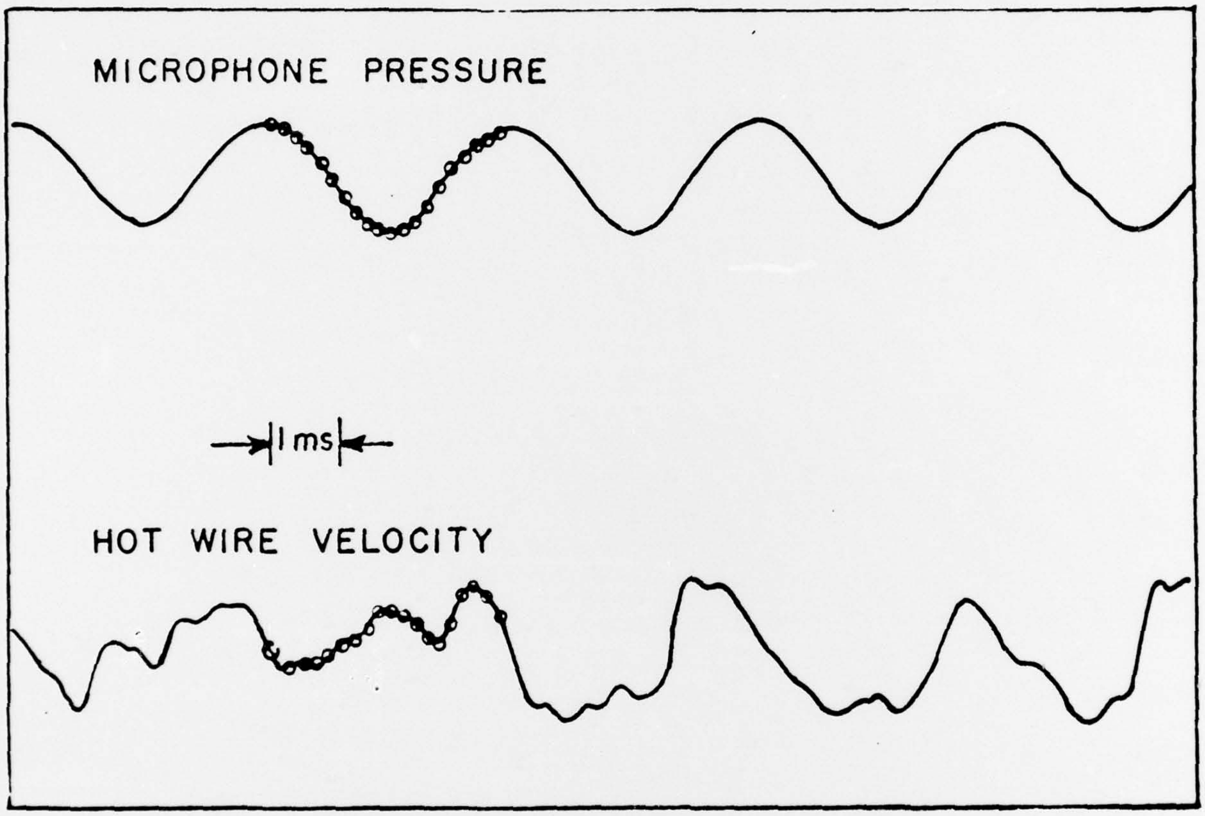


Fig 4

APPARENT PERIOD: $T' = T\{1 + \Delta f/f\}$

PHASE ERROR AT
ith CYCLE: $\Delta\theta_i = 2\pi i \Delta f/f$

INSTANTANEOUS VALUE: $A_i = A_p \cos\{\theta - \phi_1 + \Delta\theta_i\} + \varepsilon$

WHERE:

f = ESTIMATED SIGNAL FREQUENCY

Δf = ERROR IN f

A_p = "TRUE" AMPLITUDE OF PERIODIC SIGNAL

ϕ_1 = INITIAL OFFSET

ε = RANDOM NOISE COMPONENT

Fig 5

AVERAGE OVER M PERIODS

$$\overline{A} = \frac{1}{M} \sum_{-\frac{M}{2}}^{\frac{M}{2}} A_i$$

EXPANDING,

$$\overline{A} = A_p \cos\{\theta-\phi_1\} \overline{\cos \Delta\theta_M} - A_p \sin\{\theta-\phi_1\} \overline{\sin \Delta\theta_M} + \overline{\varepsilon}$$

FOR $\Delta\theta_M << 2\pi$

$$\boxed{\overline{A} \simeq A_p \cos \{\theta-\phi_1\}}$$

SINCE

$$\overline{\cos \Delta\theta_M} \simeq 1$$

$$\overline{\sin \Delta\theta_M} \simeq 0$$

$$\overline{\varepsilon} \equiv 0$$

Fig 6

STANDARD DEVIATION

$$\sigma^2 = \overline{|A_i - \overline{A}|^2}$$

SUBSTITUTING,

$$\sigma^2 \simeq A_p^2 \sin^2\{\theta-\phi_1\}\overline{\Delta\theta_M^2} - \overline{2\varepsilon\Delta\theta_M}\, A_p \sin\{\theta-\phi_1\} + \overline{\varepsilon^2}$$

OR

$$\sigma^2 \simeq A_p^2 \sin^2\{\theta-\phi_1\}\,\overline{\Delta\theta_M^2} + \sigma_r^2$$

WHERE σ_r = Std. Dev. of random component

Fig 7

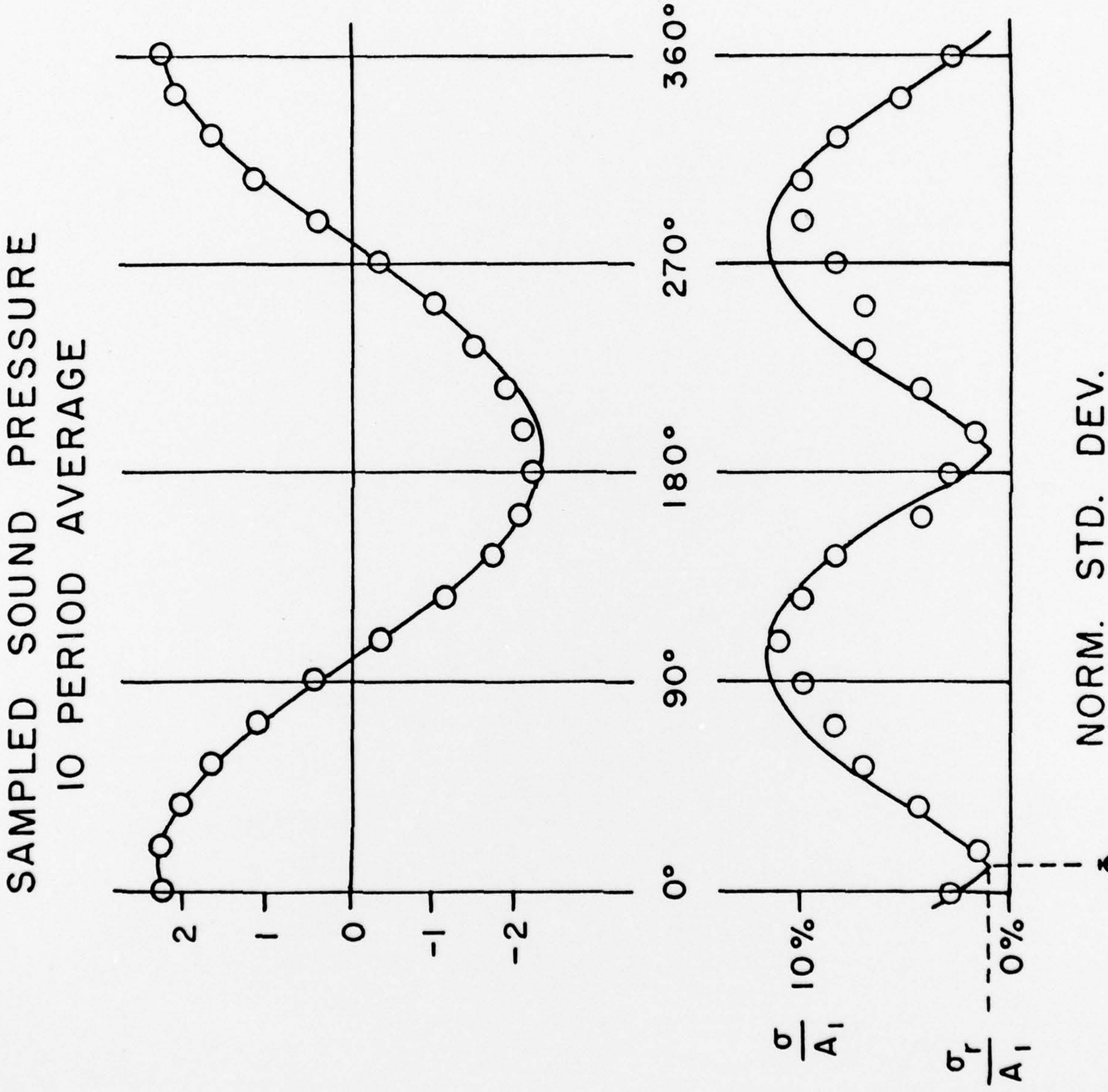
NORMALIZED STANDARD DEVIATION

$$\frac{\sigma}{A_p} = \sqrt{\sin^2\{\theta-\phi_1\}\,\overline{\Delta\theta_M^2} + \left|\frac{\sigma_r}{A_p}\right|^2}$$

WHERE

$$\overline{\Delta\theta_M^2} = \frac{1}{M-1}\sum_{-\frac{M}{2}}^{\frac{M}{2}} |2\pi i\ \Delta f/f|^2 = |2\pi\ \Delta f/f|^2 \left\{\frac{M}{M-1}\frac{\{M/2+1\}\{M+L\}}{6}\right\}$$

Fig 3



SAMPLED SOUND PRESSURE
10 PERIOD AVERAGE
2
1
0
-1
-2
0°
90°
180°
270°
360°
σ/A₁ 10%
σ_r/A₁ 0%
Φ
NORM. STD. DEV.

Fig 9

19

GENERAL THEORY: MEAN VALUE

$$A_i = \sum_{1}^{N} A_{pj} \cos\{ j\theta-\phi_j +\Delta\theta_{ij}\} + \varepsilon$$

$$\overline{A} = \sum_{1}^{N} \left[A_{pj} \cos\{j\theta-\phi_j\} \overline{\cos \Delta\theta_j} - \sin\{j\theta-\phi_j\} \overline{\sin \Delta\theta_j}\right] + \overline{\varepsilon}$$

BUT,

$$\overline{\varepsilon} \equiv 0$$

$$\overline{\sin \Delta\theta_j} = \frac{1}{M} \sum_{-\frac{M}{2}}^{\frac{M}{2}} \sin 2\pi\Delta f/f\ ij \equiv 0$$

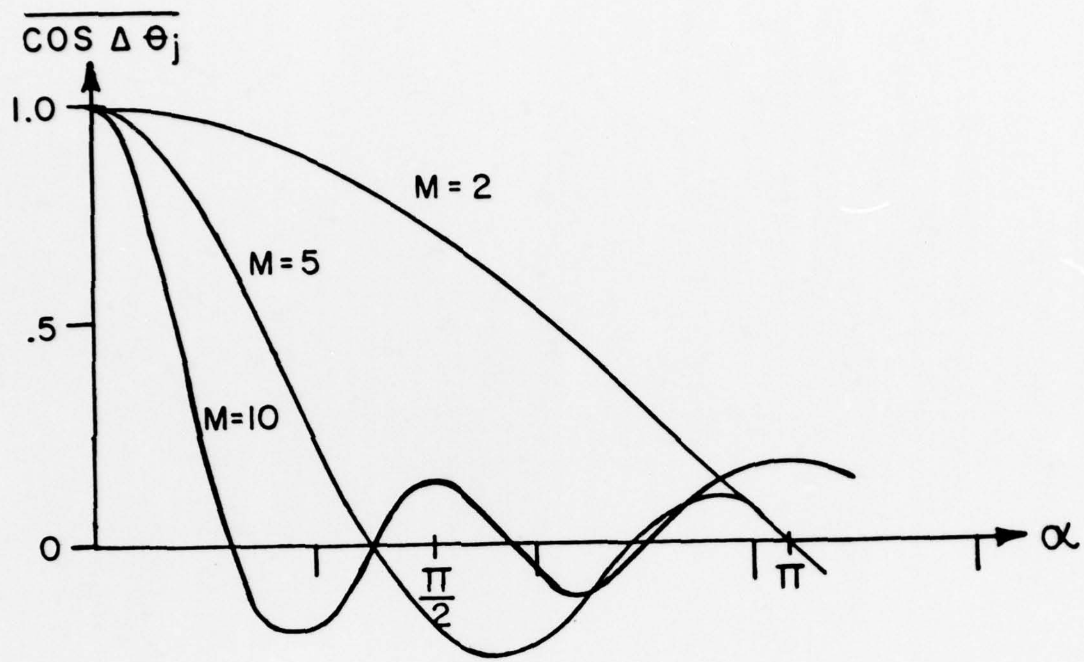
$$\overline{\cos \Delta\theta_j} = \frac{1}{M} \sum_{-\frac{M}{2}}^{\frac{M}{2}} \cos 2\pi\Delta f/f\ ij = \frac{1}{M} \frac{\sin \alpha M/2}{\sin \alpha/2}$$

WHERE $\alpha = 2\pi\Delta f/f\ j$

THEREFORE

$$\boxed{\overline{A} = \sum_{1}^{N} A_{pj} \cos\{j\theta-\phi_j\} \overline{\cos \Delta\theta_j}}$$

Fig 10



$\alpha = 2\pi \frac{\Delta f}{f} j$

Fig 11

NORMALIZED STANDARD DEVIATION

$$\sigma^2 = \sum_1^N A_{pj}^2 \left\{ \cos^2 \{j\theta - \phi_j\} \overline{\left| \cos \Delta\theta_{ij} - \overline{\cos \Delta\theta_j} \right|^2} + \sin^2 \{j\theta - \phi_j\} \overline{\sin^2 \Delta\theta_{ij}} \right\} + \overline{\varepsilon}^2$$

BUT,

$$\overline{\left| \cos \Delta\theta_{ij} - \overline{\cos \Delta\theta_j} \right|^2} \equiv 0$$

THEREFORE

$$\frac{\sigma}{A_{p1}} = \sqrt{\sum_1^N \left|\frac{A_{pj}}{A_{p1}}\right|^2 \sin^2 \{j\theta - \phi_j\} \overline{\sin^2 \Delta\theta_j} + \left|\frac{\sigma_r}{A_{p1}}\right|^2}$$

Fig 12

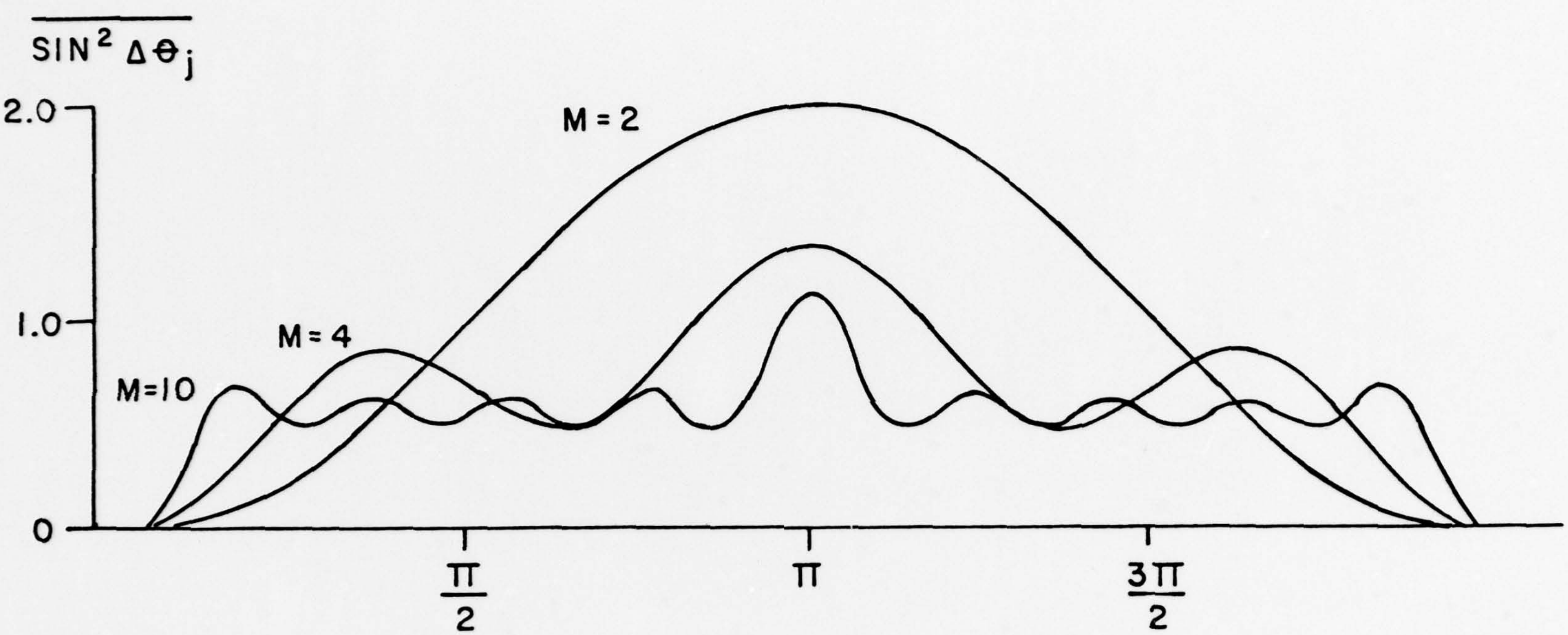
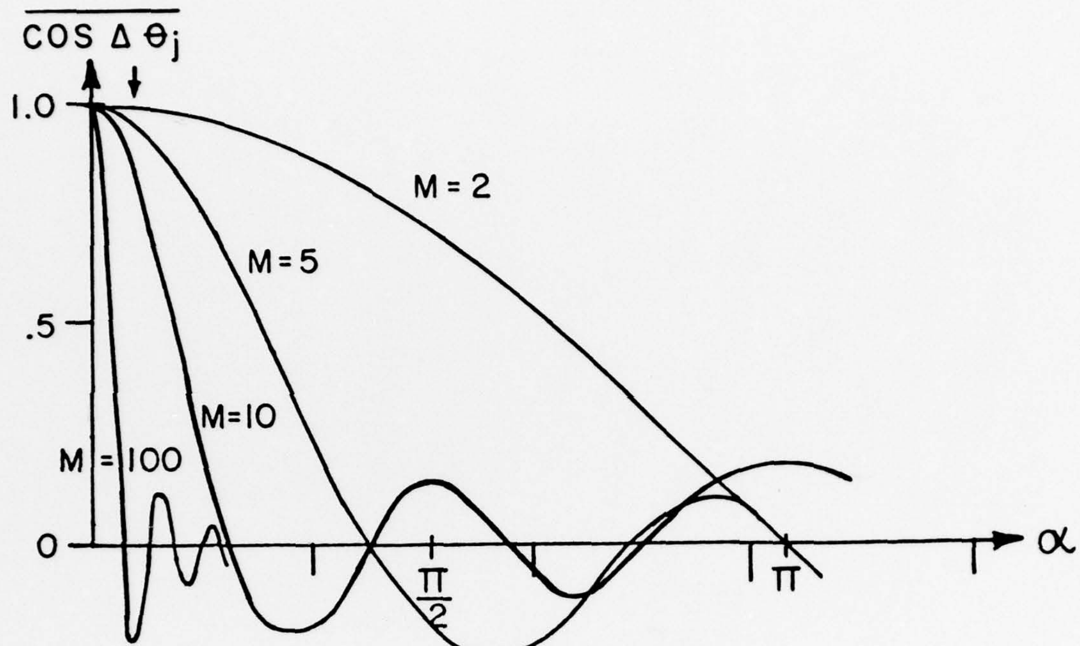

$\overline{\mathrm{SIN}^2\,\Delta\theta_j}$
2.0
1.0
0
M = 2
M = 4
M=10
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$

Fig 13



$$\alpha = 2\pi \frac{\Delta f}{f} j$$

Fig 14

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

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hot wire measurements
cash method
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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